THE OPEN UNIVERSITY OF SRI LANAKA

DIPLOMA IN TECHNOLOGY – LEVEL 03

FINAL EXAMINATION – 2010/2011

MPZ 3132 – ENGINERING MATHEMATICS IB

DURATION: THREE (03) HOURS

Date: 05th March 2011 **Time: 0930hrs – 1230hrs**

<u>Instructions</u>:

- **Answer only five (05) questions, selecting at least two(02) questions from each section A & B**
- **Number of pages in the paper – 07.**
- **Bold letters represent the vectors.**
- **All symbols are in standard notation.**
- **Last page contains the table of Normal Distribution $\boldsymbol{N(0,1)}$.**

Important integrals

- $\int e^{ax}\sin bx dx = \dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
- $\int e^{ax}\cos bx dx = \dfrac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$
- $\int f(x)e^{ax}dx = \dfrac{1}{a}f(x)e^{ax} - \dfrac{1}{a^2}f^1(x)e^{ax} + \dfrac{1}{a^3}f^2(x)e^{ax} - \ldots\ldots\ldots\ldots$

  Sign alternate $(+-+-+-\ldots\ldots\ldots)$
- $\int f(x)\cos ax dx = \dfrac{1}{a}f(x)\sin ax + \dfrac{1}{a^2}f^1(x)\cos ax - \dfrac{1}{a^3}f^2(x)\sin ax\ldots\ldots\ldots\ldots$

  Sign alternate in pairs $(++--++--++--\ldots\ldots\ldots\ldots$
- $\int f(x)\sin ax dx = -\dfrac{1}{a}f(x)\cos ax + \dfrac{1}{a^2}f^1(x)\sin ax + \dfrac{1}{a^3}f^2(x)\cos ax\ldots\ldots\ldots\ldots$

  Sign alternate in pairs after the first term $(++--++--++--\ldots\ldots\ldots\ldots$
- $\int \sec ax dx = \dfrac{1}{a}\ln\left|\sec ax + \tan ax\right|$

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SECTION – A

1. The function f is defined as $f(x) = x^2 \quad x \in (0\,,1)$

   a) Extend the above function as a

      (i) Odd periodic function with period 2

      (ii) Even periodic function with period 2

      (iii) Periodic function with period 1

   Marks 40

   b) Draw the graphs of the above three functions on $[-3\,,3]$

   Marks 30

   c) Find the Fourier representation of $f(x) = x^2 \quad x \in (0\,,1)$

      (i) As a sine series with period 2

      (ii) As a cosine series with period 2

      (iii) As a full trigonometric series with period 1

   Marks 130

2. (a) Define the Taylor series expansion of $f(x)$ about $x = a$.

   Prove that Taylor series expansion of $\frac{1}{x}$ about $x = 3$ is $\sum_{r=0}^{\infty} \frac{(-1)^r (x-3)^r}{3^{r+1}}$.

   Show also that the above series converges to $\frac{1}{x}$ and find the radius of convergence.

   (Hint: Use the convergence of a geometric series.)

   Marks 70

   (b) If $\boldsymbol{r}\,, \boldsymbol{v}$ and $\boldsymbol{a}$ are the position vector, velocity and acceleration of a particle at time t respectively.

   Prove that $\frac{d}{dt}(\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})) = 2(\boldsymbol{v} \cdot \boldsymbol{a})\boldsymbol{r} - (\boldsymbol{r} \cdot \boldsymbol{a})\boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{r})\boldsymbol{a}.$

   Marks 50

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(c) Prove that if $\boldsymbol{f}(t) = \frac{e^t sint}{\sqrt{1+e^{2t}}}\mathbf{i} + \frac{e^t cost}{\sqrt{1+e^{2t}}}\mathbf{j} + \frac{1}{\sqrt{1+e^{2t}}}\mathbf{k}$

(i) $|\boldsymbol{f}(t)| = 1$

(ii) $\boldsymbol{f}(t)$ is perpendicular to $\frac{d}{dt}(\boldsymbol{f}(t))$

(iii). $\boldsymbol{f}(t) \cdot \frac{d^2(\boldsymbol{f}(t))}{dt^2} = -\left|\frac{d(\boldsymbol{f}(t))}{dt}\right|^2$

Marks 80

3. (a) The times taken to germinate for a certain kind of paddy seeds are normally distributed. If 20% of the seeds take more than six days to germinate and if 10% of the seeds germinate in less than 4 days, then find the mean and the standard deviation of the time for germination of the paddy seeds.

Marks 140

(b) The height of a large batch of a certain crop are normally distributed with a mean of $140\ cm$ and a standard deviation of $6cm$.

(i) Find the probability that the height of a randomly selected tree of that batch is less than $145\ cm$.

(ii) If five trees are selected at random from the above batch ,find the probability that all the five trees are less than $145\ cm$ tall and only three of them are less than $145\ cm$.

Marks 60

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4. (a) A particle of mass m is projected vertically upwards with velocity u in a resistive medium whose resistance is mkv^2 ,where v is the speed of the particle and k is a constant. Prove that the greatest height reached by the particle is $\frac{1}{2k} ln\left(1 + \frac{ku^2}{g}\right)$

Marks 60

(b)A uniform rod of mass m and length 2a can turn freely about one end which is fixed. It is given an angular velocity ω from the position in which it hangs vertically. Show that if θ is the inclination of the rod to the down-ward vertical at time t, then

$\dot{\theta}^2 = \omega^2 - \frac{3g}{2a}\ (1 - cos\theta)$.

Find the least value ω_0 of ω so that the rod may just make complete revolutions. With this particular angular velocity ω_0, show that the time of describing any angle $\theta(< \pi)$ is

$2\sqrt{\frac{a}{3g}} ln\left[tan\left(\frac{\pi+\theta}{4}\right)\right]$.

Marks 140

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SECTION – B

5. (a) Using a method of trial function find a particular solution of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 34e^{3x}$. Hence find the general solution of the above differential equation.

Marks 65

(b) Prove that $\frac{1}{D+\alpha} f(x) = e^{-\alpha x}\frac{1}{D}e^{\alpha x}f(x)$.

Using the above formula find the particular integral of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 5sin3x$ .Hence find the general solution of the above differential equation.

Marks 135

6. (a) Shade the region satisfying the inequality $Re(z+2) \geq |z-2|$ and $|z| \leq 2$.

Marks 70

(b) Prove that $Log(1+i) = \frac{1}{2}ln2 + i(2k+\frac{1}{4})\pi$ where $k \in Z$.

Marks 45

(c) If $f(z) = \frac{4z+3}{z-1}$ where $z \in C\backslash\{1\}$, then show that f is a one to one function and find the inverse function of f.

Marks 25

(d) If the function g is defined as $g(z) = z^2$, $|z| \leq 2$ and $Im(z) \geq 0$,then find the image of the function and shade the domain and the range of g.

Marks 60

. Please Turn Over

7. (a) Using the augmented matrix and elementary row operations ,discuss the solution of the following system of equations .When the system has solutions ,find those solutions.

$$x + 2y - 3z = 4$$

$$3x - y + 5z = 2$$

$$4x + y + (\mu^2 - 14)z = \mu + 2 \quad \text{Where } \mu \in R$$

Marks 125

(b) Find the solution of the following differential equation using Laplace transformation.

$\frac{d^2y}{dx^2} + y = 3sin2x$, where $x \geq 0$, $y(0) = 1$ and $\left(\frac{dy}{dx}\right)_{x=0} = 2.$

Marks 75

8. A uniform rectangular lamina $ABCD$ is immersed vertically in a homogeneous liquid such that AB and CD are horizontal. The distances to AB and CD from the surface of the liquid are h_1 and h_2 respectively. Prove that that the centre of pressure of the lamina is at the distance

$$\frac{2}{3}\left(\frac{h_1^2 + h_1h_2 + h_2^2}{h_1 + h_2}\right)$$

from the surface.

Marks 120

The inside cross section of a closed right cone is a square of side $2a$ and the height of the inside of the cone is h. The cone is completely filled with a homogeneous liquid. The weight of the liquid inside the cone is w . If the cone is kept such that the axis of the cone is horizontal, find the reaction on the angled surfaces of the cone.

Marks 80

END